

[Answer key]

ACT 2020, MIDTERM #1  
ECONOMIC AND FINANCIAL APPLICATIONS  
FEBRUARY 8, 2007  
HAL W. PEDERSEN

You have 70 minutes to complete this exam. When the invigilator instructs you to stop writing you must do so immediately. If you do not abide by this instruction you will be penalised. All invigilators have full authority to disqualify your paper if, in their judgement, you are found to have violated the code of academic honesty.

Each question is worth 10 points. Provide sufficient reasoning to back up your answer but do not write more than necessary.

This exam consists of 8 questions. Answer each question on a separate page of the exam book. Write your name and student number on each exam book that you use to answer the questions. Good luck!

**Question 1.** Bill and Jim enter into a binding contract involving the exchange of an asset in nine months time. The current market price of the asset is \$100. The continuously compounded interest rate is  $r = 0.045$ . The contract calls for Jim to sell the asset to Bill at that time at a price  $G$  that is set today. The price  $G$  is set so that a fair valuation of the contract requires Jim to pay Bill \$2.20 today. If the price of the asset in nine months time is \$98 what is Jim's total profit or loss at the time the asset is sold?

**Question 2.** John has just purchased a home for \$200,000. The insurance policy will cover any losses due to fire subject to a deductible of \$10,000. The one-year premium on the insurance policy is \$3,000 which is due at the start of the policy year.

(1) [5 points] Let  $M$  denote the market value of John's home at the end of the policy year.<sup>1</sup> Draw a chart of the net value of John's home and insurance policy versus  $M$  at the end of the policy year. Assume  $r = 0$  for simplicity.

(2) [3 points] Explain in what sense there is a put option implicit in this transaction, possibly drawing an appropriate chart.

(3) [2 points] Explain in what sense there is a call option implicit in this transaction, possibly drawing an appropriate chart.

---

<sup>1</sup>If there is a devastating fire the market value of John's home could be close to zero. If there is no fire and the real estate market soars then the market value of John's home could be more than John's purchase price of \$200,000.

**Question 3.** Your client currently owns one share of XYZ Corporation and has an investment horizon of six months. The current market price of one share of XYZ Corporation is \$25. Your client is considering writing one European call option on XYZ Corp. that expires in six months. The market price of an at-the-money European call option expiring in six months is \$2.35. The market price of a European call with a strike price of \$27.50 expiring in six months is \$1.35.

(1) [3 points] Draw a chart showing the total profit at the end of six months from holding one share of XYZ Corporation and writing one at-the-money European call option expiring in six months as a function of the XYZ Corporation share price in six months time. (Your chart should provide numerical values identifying the key parts of the chart as was done in class.)

(2) [4 points] Draw a chart showing the total profit at the end of six months from holding one share of XYZ Corporation and writing one European call with a strike price of \$27.50 expiring in six months as a function of the XYZ Corporation share price in six months time. (Your chart should provide numerical values identifying the key parts of the chart as was done in class.)

(3) [3 points] What is the greatest possible total return at the end of six months from holding one share of XYZ Corporation and writing one European call with a strike price of \$27.50 expiring in six months?

**Question 4.** Assume that you open a 100 share short position in Jiffy Inc. common stock at the bid-ask price of \$32.00 - \$32.50. When you close your position the bid-ask prices are \$32.50 - \$33.00. If you pay a commission rate of 0.5%, calculate your profit or loss on the short investment?

**Question 5.** The Federated Bank of Canada is offering a structured product that guarantees the return of 85% of the investor's capital at the end of 5 years. The continuously compounded interest rate is  $r = 0.05$ . The index underlying the structured product is currently at 1200 (*i.e.*  $S_0 = 1200$ ). The current market price of one at-the-money European call on the underlying index expiring in 5 years is \$350.

(1) [5 points] Assuming that the Federated Bank of Canada charges the investor nothing for this contract (*i.e.* the haircut is zero), compute the participation rate for the structured product.

(2) [5 points] Assume that the Federated Bank of Canada tells the investor that after their fee is applied the investor's total return for the five year investment horizon has the profile

$$R = -0.15 + 0.9875 \cdot (R_{\text{INDEX}})_+$$

As in class,  $R_{\text{INDEX}}$  denotes the total return on the underlying index for the five-year period. What has the investor been charged for this product as a percentage of their initial investment?



**Question 6.** The continuously compounded interest rate is  $r = 0.08$ . The market price of a European put option on a market index that expires in 4 months is \$93.39348. The current level of the market index is 1100 and the strike level on the put option is 1200. What is the price of a European call option on a market index that expires in 4 months with a strike level of 1200?

Question 7. [Practice Problems Chapter 2 #6]

Suppose the 6-month \$1,200-strike S&R index put option trades at a bid of \$72.97 and at an ask of \$74.44. Suppose you can enter into an S&R index forward contract for \$1,260. The 6-month effective interest rate is 5%.

- [5 pts] (a) Draw, in the same diagram, the profit for a short put and a long forward position.
- [5 pts] (b) At what price is the profit of the put option the same as the profit of the forward contract? If the index level is higher than the threshold you calculated, which instrument (put or forward) makes more money?

Question 8. [Text 2.14]

Suppose the stock price is \$40 and the effective annual interest rate is 8%. Draw payoff and profit diagrams for the following options:

- 35-strike put with a premium of \$1.53.
- 40-strike put with a premium of \$3.26.
- 45-strike put with a premium of \$5.75.

(1)

Q1: Approach 1:Long CF:  $S_T - G$  ( $T = 6$  months)

$$-2.20 = \text{Value of Payment of } S_T - G \text{ at } T$$

$$= S_0 - e^{-rT}G$$

$$100 - 2.20 = 100 - e^{-.045(.75)}G$$

$$\Rightarrow G = 105.71$$

[ Bill gets the CF  $S_T - G$  + 2.20 today to accept the deal.  $\therefore$  the CF  $S_T - G$  must have -2.20 as its fair value. ]

Jim's Profit:

$$-2.20 e^{.045(.75)} + 105.71 - 98$$

$$= 5.43$$

Answer = \$5.43

Q1 (cont'd):

Approach 2:

$$S_T - 6 = \underbrace{S_T - F}_{\text{Forward CF}} + \underbrace{F - 6}_{\text{Deterministic CF}}$$

$$\begin{aligned} \circ \quad -2.20 &= \text{Fair Value " } S_T - 6 \text{"} \\ &= \text{Value Forward} + e^{-rT}(F - 6) \end{aligned}$$

$$-2.20 = e^{-rT}(F - 6)$$

$$\circ \quad -2.20e^{rT} = F - 6 \quad \left[ \begin{array}{l} -2.20e^{rT} + 6 \\ = F \end{array} \right]$$

Jim's Profit

$$\begin{aligned} &= -2.20e^{rT} + 6 - S_T \\ &= F - S_T \\ &= e^{.045(.75)} 100 - 98 \\ &= 5.43 \end{aligned}$$

More Simply:

Jim pays Bill the \$2.20 today to receive  $6 - F$  in 6 months.

$$\text{[i.e. } 2.20 = e^{-rT}(6 - F)\text{]}$$

The residual contract is a forward.  $\circ$  with the "side payment" removed, Bill & Jim are in a forward contract.



Q2:  $V \equiv$  net value of John's home and insurance policy

(1)  $=$  Market Value of Home  $+$  Payment from Policy  $-$  Cost of Policy  
OR Fut. Value Cost if  $r \neq 0$

Market Value of Home  $= M$

Payment from Policy  $^* = (190,000 - M)_+$

Cost of Policy  $= 3,000$

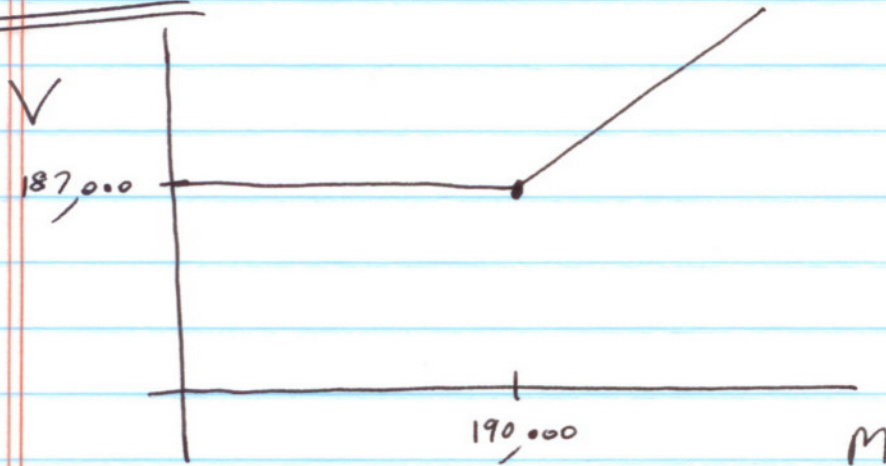
$V = M + (190,000 - M)_+ - 3,000$

$= \begin{cases} 187,000 & M \leq 190,000 \\ M - 3,000 & M > 190,000 \end{cases}$

---

\*  $(190,000 - M)_+ = (200,000 - [(200,000 - L) - 10,000])_+$   
M

Q2 (cont'd):



You can get the chart by general reasoning instead of a mathematical equation.

(2) Payoff from insurance policy is a put option on market price/value of house with a strike price of 190,000.  
 $[ 190,000 = \text{Insured Value} - \text{Deductible} ]$   
 $= 200,000 - 10,000 ]$

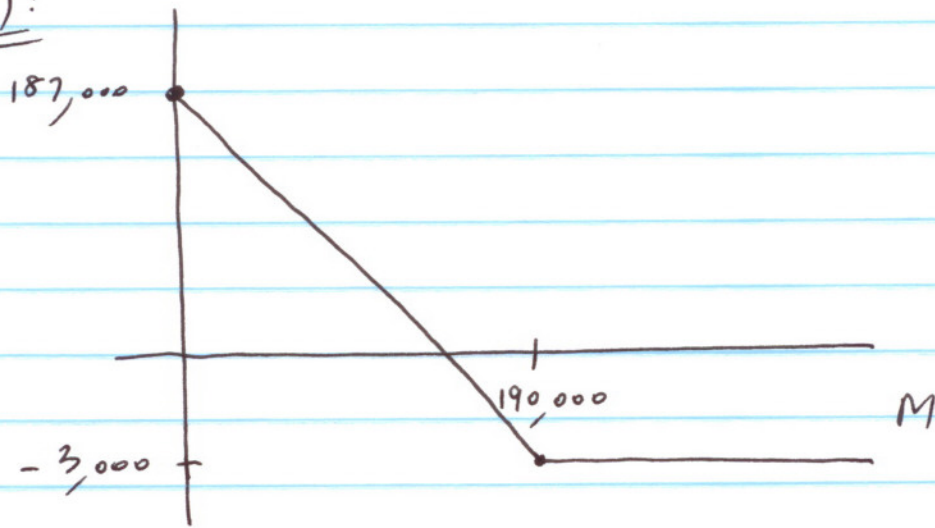
If you include the premium, the total "profit" from buying the policy is:

$$(190,000 - M) + - 3,000$$

This is also a reasonable explanation but in either case the put is in the policy payment.

Q2 (cont'd):

Profit



- (3) As discussed on pg. 62 of the text, "an insured house has a profit diagram that looks like a call option".

Mathematically, the net value (as asked in (i) above) is:

$$\begin{aligned}
 V &= M + (190,000 - M)_+ - 3000 \\
 &= 190,000 + (M - 190,000)_+ - 3000 \\
 &= 187,000 + (M - 190,000)_+
 \end{aligned}$$

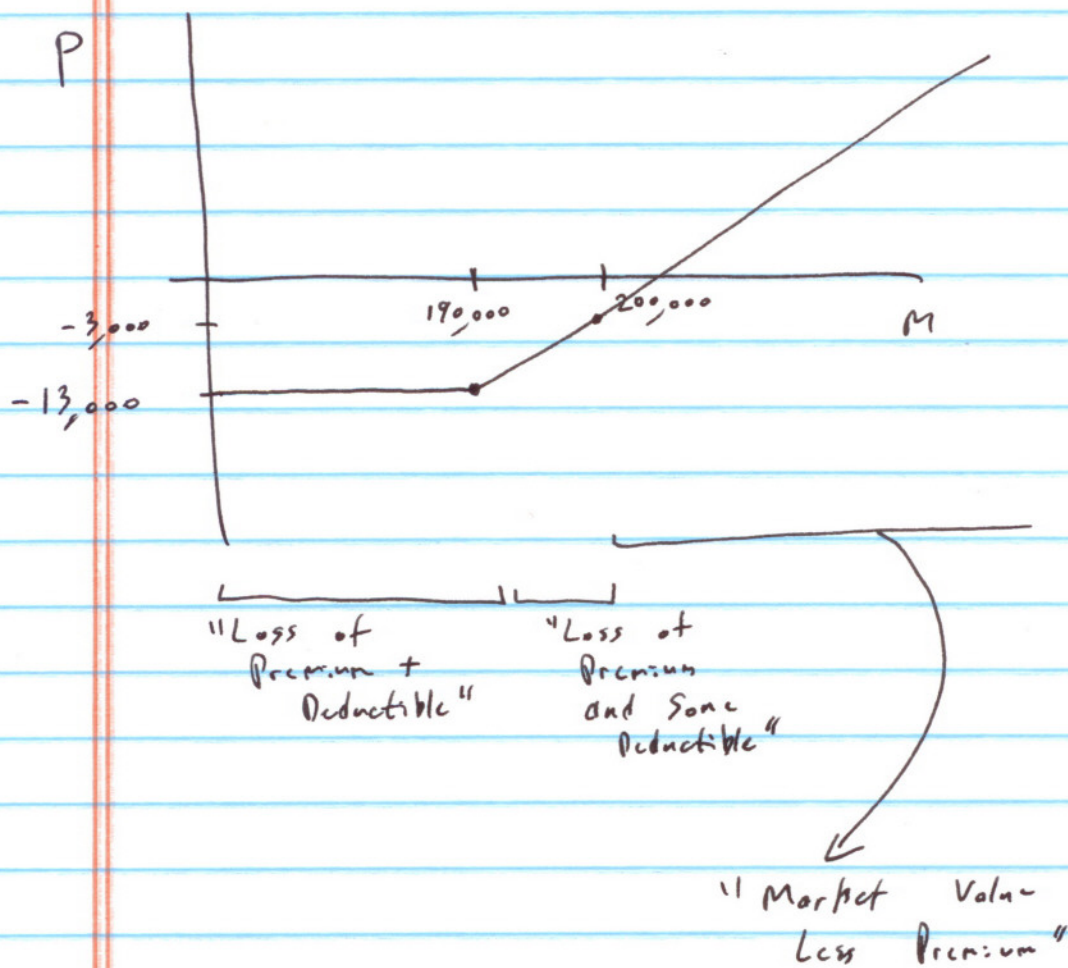
∴ the market value of the net position is 187,000 plus a call on the market value of the home with a strike price of 190,000.



Q2 (cont'd):

If one deducts the cost of the house one gets the total profit:

$$P = -13,000 + (M - 190,000) +$$



Key is to recognise there is a call on market value with strike of 190,000.

Q3: Assume  $r=0$  since no value given.  
 (If you assume another value for  $r$  that is also fine.)

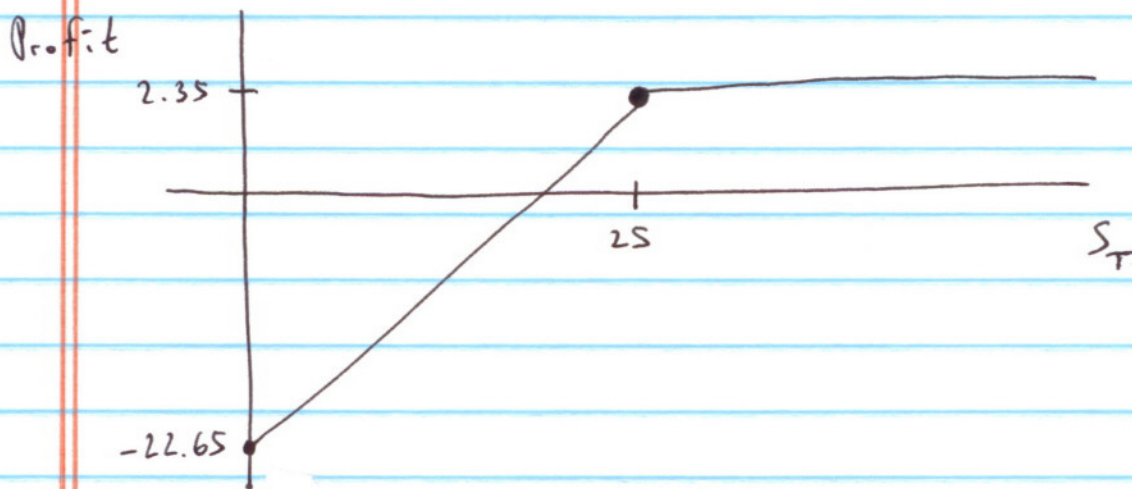
$S_T = x_T$  share price at end of 6 months

$$\text{Profit} = S_T - S_0 - (S_T - 25)_+ + 2.35$$

$$= S_T - 25 - (S_T - 25)_+ + 2.35$$

$$= 2.35 - (25 - S_T)_+ \quad *$$

$$= \begin{cases} S_T - 22.65 & S_T \leq 25 \\ 2.35 & S_T > 25 \end{cases}$$



\*

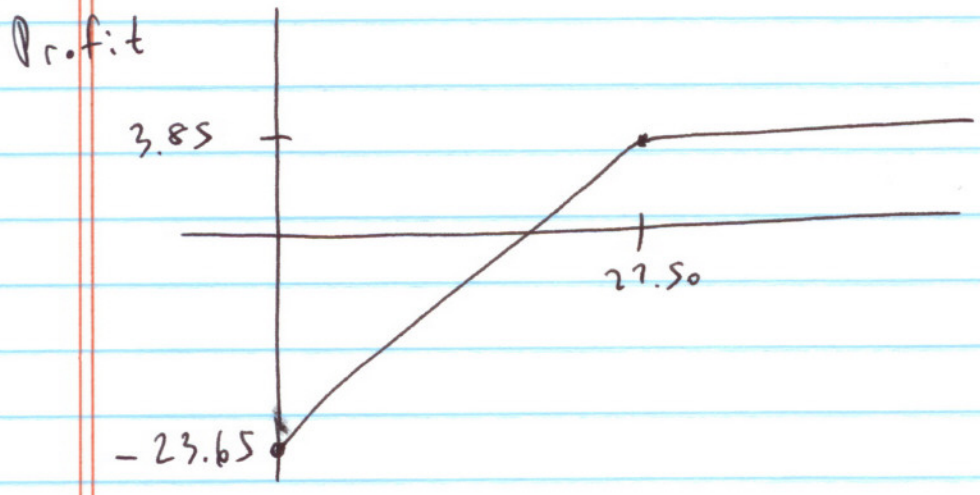
$$(S_T - 25)_+ - (25 - S_T)_+ = S_T - 25$$



Q3 (cont'd): If you forgot to include cost of XYZ stock the chart shifts up \$25.

$$\begin{aligned}
 (2) \text{ Profit} &= S_T - S_0 - (S_T - 27.50)_+ + 1.35 \\
 &= S_T - 25 - (S_T - 27.50)_+ + 1.35 \\
 &= S_T - 27.50 - (S_T - 27.50)_+ + 3.85 \\
 &= 3.85 - (27.50 - S_T)_+
 \end{aligned}$$

$$= \begin{cases} S_T - 23.65 & S_T \leq 27.50 \\ 3.85 & S_T > 27.50 \end{cases}$$



$$(3) \quad \text{Greatest Return} = \frac{\text{Highest Payoff} - \text{Cost of Position}}{\text{Cost of Position}}$$

$$= \frac{\text{Highest Payoff}}{\text{Cost of Position}} - 1$$

$$= \frac{28.85}{25 - 1.35} - 1 = .2199$$

$$\text{Answer} = 22\%$$

Alternative:

$$\text{Greatest Return} = \frac{\text{Ending Value} - \text{Beginning Value}}{\text{Beginning Value}}$$

$$= \frac{28.85 - (25 - 1.35)}{25 - 1.35} = .22$$

Q4. Revenue from Short Sale =  $(1 - .005) 100 (32) = 3184$

Cost to Cover =  $(1 + .005) 100 (33) = 3316.50$

Loss =  $3316.50 - 3184 = 132.50$

$$\text{Answer} = \text{Loss of } \$132.50$$



Q5: $I_0 =$  initial investment

$$(1) \quad .85 I_0 = B e^{.05(s)}$$

$$\Rightarrow B = e^{-.05(s)} \cdot .85 I_0$$

∴  $I_0 - e^{-.05(s)} \cdot .85 I_0$  into calls.

$$CF = .85 I_0 + \frac{I_0(1 - e^{-.05(s)} \cdot .85)}{350} (S_5 - 1200)_+$$

$$= \left[ .85 + \frac{(1 - e^{-.05(s)} \cdot .85)}{350} 1200 \left( \frac{S_5}{1200} - 1 \right)_+ \right] I_0$$

$$\therefore R = \frac{CF}{I_0} - 1$$

$$= -.15 + (1 - e^{-.05(s)} \cdot .85) \frac{1200}{350} (R_{Index})_+$$

$$\text{Participation Ratio} = (1 - e^{-.05(s)} \cdot .85) \frac{1200}{350}$$

$$= 1.1589$$

$$\text{Answer} = 1.1589$$

(2) If  $(1-\beta)\%$  capital guaranteed:

$$(1-\beta)I_0 = e^{rT} B.$$

$h$  = haircut as % of  $I_0$ ,  
 $\therefore$  haircut =  $h I_0$ .

Amount for ATM Coll =  $I_0 - e^{-rT}(1-\beta)I_0 - hI_0$ .

$$CF = (1-\beta)I_0 + \frac{[1 - (1-\beta)e^{-rT} - h]I_0 (S_T - S_0)_+}{C}$$

$$R = \frac{CF}{I_0} - 1$$

$$= -\beta + [1 - (1-\beta)e^{-rT} - h] \frac{S_0 (R_{Index})_+}{C}$$

$$\therefore .9875 = [1 - .85 e^{-.05(5)} - h] \frac{12.00}{35.0}$$

$$\Rightarrow h = .05$$

Answer = 5%



Q6:

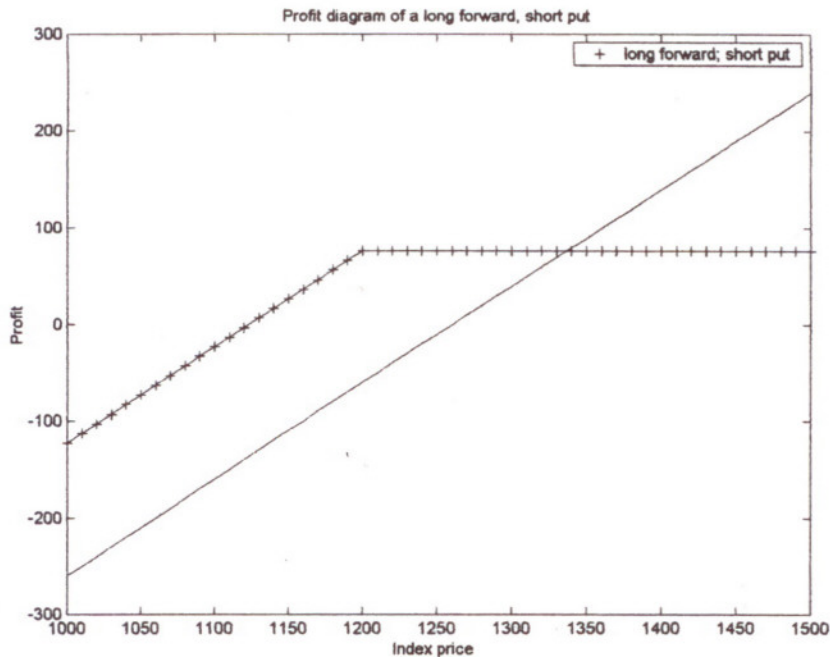
$$C - P = S_0 - e^{-rT} K$$

$$C = 93.39348 + 1100 - e^{-.08(1/3)} 1200$$
$$= 24.97$$

Answer = 24.97

Q7:

(a)



(b) We need to solve (remember that we receive the proceeds from the sale of the put option, and can earn interest on them):

$$72.97 \times (1 + 0.05) = S_T - \$1,260$$
$$\Leftrightarrow S_T = \$1,336.62$$

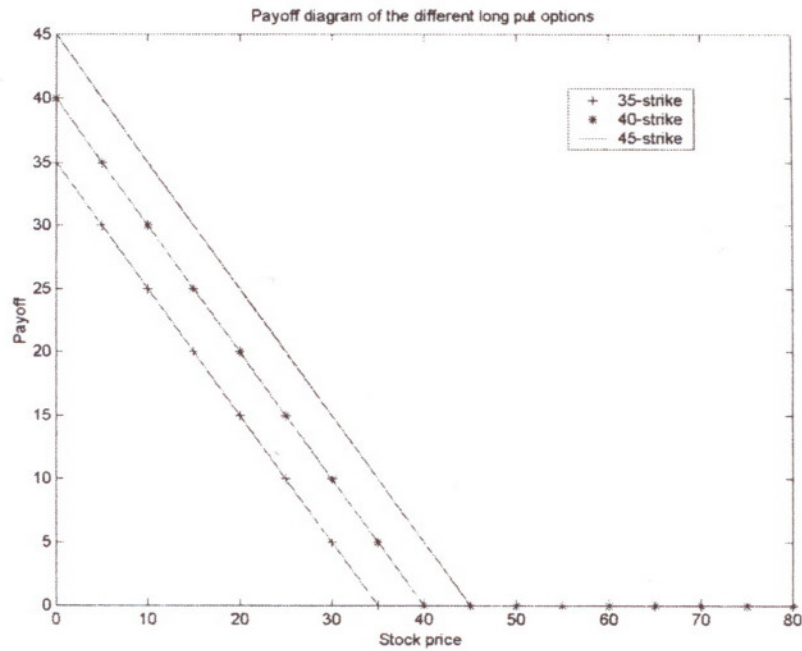
Our profit from the sold put option is capped at the initial premium, plus interest, that we receive. Therefore, to the right of the threshold, we earn more profit with the long forward contract.

Q8:

In order to be able to draw profit diagrams, we need to find the future values of the put premia. They are:

- a) 35-strike put:  $\$1.53 \times (1 + 0.08) = \$1.6524$
- b) 40-strike put:  $\$3.26 \times (1 + 0.08) = \$3.5208$
- c) 45-strike put:  $\$5.75 \times (1 + 0.08) = \$6.21$

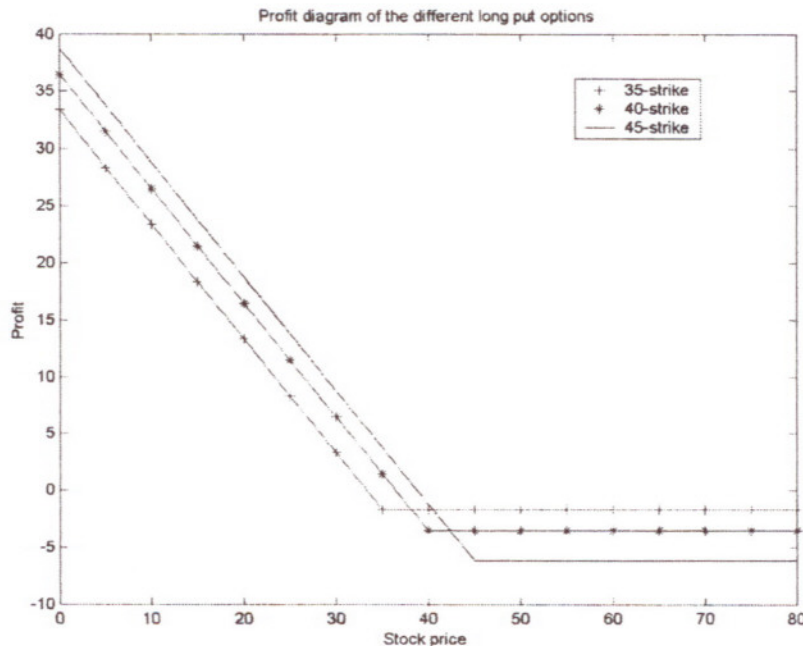
We get the following payoff diagrams:





Q8 (cont'd):

We get the profit diagram by deducting the option premia from the payoff graphs. The profit diagram looks as follows:



Intuitively, whenever the 35-strike put option pays off (i.e., has a payoff bigger than zero), the 40-strike and the 35-strike options also pay off. However, there are some instances in which the 40-strike option pays off and the 35-strike options does not. Similarly, there are some instances in which the 45-strike option pays off, and neither the 40-strike nor the 35-strike pay off. Therefore, the 45-strike offers more potential than the 40- and 35-strike, and the 40-strike offers more potential than the 35-strike. We pay for these additional payoff possibilities by initially paying a higher premium. It makes sense that the premium is increasing in the strike price.